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Influence of the angular response on Fourier absolute spectrometry the case of COBE-FIRAS

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Abstract

The calibration procedure in FFT absolute spectrometry requires a comparison between two or more reference sources. The influence of the angular response of the corresponding apertures is investigated: a difference of the order of 5% between the solid angles of the two optics of FIRAS is assumed and the consequences on the new upper limits on μ and y are studied. It results that the upper limit on μ is reasonable, while the upper limit on y is not significative. The effects on temperature measurement are also studied, but they are negligible compared to the systematic error given by FIRAS team.

1. Introduction

Since the discovery of the cosmic microwave background radiation (CMBR), one of the most important goals of experimental cosmology has been to measure its spectrum with increasing accuracy. The CMBR spectrum is expected to have a black-body shape if the early universe is correctly described by the simple hot big-bang model, yet this shape could be distorted by energy release after a redshift $z \sim 3 \times 10^6$. The shape of possible distortions has been computed by many authors (Peebles [1]; Sunyaev and Zel'dovich [2]). From a quantitative point of view such distortions can be described by two dimensionless cosmological parameters: the chemical potential μ of the Bose–Einstein distribution (energy release in the red-shift range $3 \times 10^6 > z > 10^5$) and the Comptonization parameter, y, which appears in the Kompaneets equation (energy release in the redshift range $z < 10^5$).

FIRAS (Far Infrared Absolute Spectrometer) instrument on the COBE satellite (see Mather et al. [3]; Boggess et al. [4]) was designed to measure very small deviations of the CMBR from the blackbody spectrum with higher accuracy and sensitivity than previously. The new results from FIRAS (see Mather et al. 1994 [5]) place tight limits on distortions of the CMBR and give new upper limits on μ and y. The FIRAS basic instrument is a

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Michelson interferometer employed as a zero-instrument: the difference between the power received from the sky and that from an internal reference calibrator is minimized.

In this paper the dependence of an interferometer output signal on the solid angle of the optics is analysed to discuss the zero-system employed by FIRAS. It will be proved that the new upper limit on y is below the error introduced by a reasonable difference ($\sim 5\%$) between the two solid angles of the two optics of FIRAS.

2. Analysis of the solid angle difference

If the path difference between the two arms of an interferometer is x, the power received $W(x)$ is proportional to

$$\int_0^{\infty} S(\nu)\left[1+\cos\left(4\pi\nu x\left(1-\frac{\Omega}{4\pi}\right)\right)\right]\mathrm{d}\nu, \qquad (1)$$

where $S(\nu)$ is the source brightness, Ω is the solid angle of the optics and the frequency is expressed in cm^{-1}.

The FIRAS optics are symmetrical, with two input and two output ports: one input port receives emission from the sky, the other input port receives emission from an internal reference calibrator. When observing the sky, the spectrometer is operated with its output nearly nulled and with the on-board reference calibrator adjusted to match the sky temperature. Therefore, the power $W(x)$ to be minimized is proportional to (see also Melchiorri and Melchiorri (1994) [6]):

$$\int_0^{\infty}\left\{CB(\nu)\left[1+\cos\left(4\pi\nu x\left(1-\frac{\Omega_{\mathrm{sky}}}{4\pi}\right)\right)\right] - \mathrm{ICAL}(\nu)\left[1+\cos\left(4\pi\nu x\left(1-\frac{\Omega_{\mathrm{ref}}}{4\pi}\right)\right)\right]\right\}\mathrm{d}\nu \qquad (2)$$

where a difference between the two solid angles of the optics has been considered. Such a difference could be due just to the features of instruments of this kind: it is possible to know with high accuracy the FWHM, but not the whole angular responsivity. Therefore, the solid angles of the optics have not a well-defined value and there could be a difference between them of the order of 5%, due to unknown border effects. Moreover, constructive limitations could be considered: the reference horn is much smaller than the sky horn because of spacecraft limitations, and the same effective length-to-diameter ratio could have been difficult to realize with the high accuracy that was necessary to place the new stringent limits on μ and y. FIRAS systematic errors have been reduced by a second reference calibrator that, when in place, fills the entire aperture of the sky horn. The details of calibration can be found in Fixsen et al. (1994) [7]. It is not clear, however, if the use of the external reference calibrator can rectify the error due to a difference between the two solid angles: when the external calibrator is in place it seems that the solid angle of the optics is not just the same as when the sky is being observed. In Fixsen's et al. paper [7] only the error due to a difference between the étendues has been considered and corrected, that is why in Eq. (2) the étendues have been supposed similar and inserted in the multiplicative constant.

Even if the brightness of the two sources is the same, the difference assumed between the two solid angles introduces a non-zero signal, the Fourier transform of which is the error to be considered:

$$\Delta I(\nu) = \mathrm{BB}(\nu, T) - \frac{1-\Omega_{\mathrm{sky}}/4\pi}{1-\Omega_{\mathrm{ref}}/4\pi} \times \mathrm{BB}\left[\left(\frac{1-\Omega_{\mathrm{sky}}/4\pi}{1-\Omega_{\mathrm{ref}}/4\pi}\right)\nu, T\right], \qquad (3)$$

where $\mathrm{BB}(\nu, T)$ stands for the pure blackbody spectrum at temperature T. Assuming $\Omega_{\mathrm{sky}} = \Omega_{\mathrm{ref}} \pm \Delta\Omega$, a linear approximation with respect to the parameter $q = \Delta\Omega/(4\pi - \Omega)$ gives

$$\Delta I(\nu) = q\left(\mathrm{BB}(\nu, T) + \nu\,\frac{\partial \mathrm{BB}(\nu, T)}{\partial \nu}\right), \qquad (4)$$

where only small deviations from the blackbody spectrum are considered.

FIRAS has a spectral resolution of $\sim 0.8\%$ (see Fixen et al. [7]), limited by the beam divergence. From this value it is possible to make an estimate of the solid angle of $\sim 4\times 10^{-2}$ sr. Assuming a difference of 5% between the two solid angles, the corresponding value of q is $\sim 2\times 10^{-4}$, so the linear approximation is possible (Fig. 1).

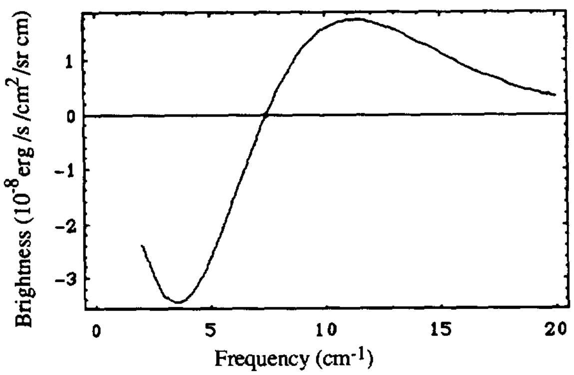

Fig. 1. Error due to a 5% difference between the two solid angles (in the same units the peak brightness is 1.2×10^4). The pure blackbody spectrum corresponds to the x-axes.

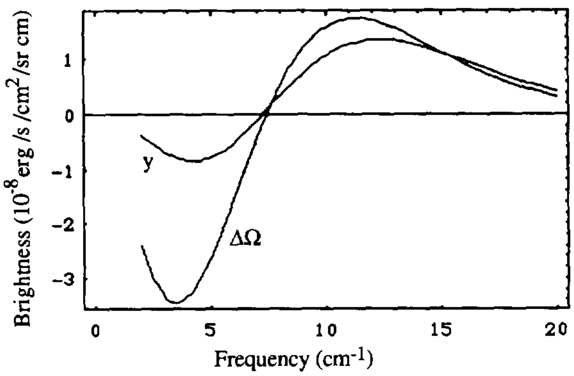

Fig. 2. Error due to a 5% difference between the two solid angles compared to the Compton distortion with $y = 2.5 \times 10^{-5}$. The former masks the latter almost completely. The pure blackbody spectrum corresponds to the x-axes.

The data from FIRAS (Mather et al. [5]) exclude any spectral deviation of the CMBR from the blackbody shape larger than 0.03%, with an rms value of 0.01%. This constraint places an upper limit on chemical poential of $|\mu| < 3.3 \times 10^{-4}$, and a much better one on the Comptonization parameter, $y < 2.5 \times 10^{-5}$, both with a 95% confidence level. To get such stringent limits Mather et al. [5] have performed a linear fit

$$I(\nu) = \mathrm{BB}(\nu, T_0) + \Delta T \frac{\partial \mathrm{BB}}{\partial T} + G_0 g(\nu) \tag{5}$$

to the unknown parameters G_0, and ΔT, where $I(\nu)$ is the monopole spectrum obtained from the data after the dipole component and the Galactic emission component had been eliminated, the first two terms on the r.h.s. are the Planck spectrum with temperature $T_0 + \Delta T$ and the third one allows for an additional Galactic contribution. Then an additional term is inserted to fit the residual deviations to one of the cosmological distortion parameters (either μ or y). Since the deviations are very small only linearized models are used, so we have for the Bose–Einstein distortion:

$$\Delta I(\nu) = \mu \frac{(-T_0)}{x} \frac{\partial \mathrm{BB}}{\partial T} \tag{6}$$

and for the Compton distortion:

$$\Delta I(\nu) = y T_0 \frac{\partial \mathrm{BB}}{\partial T} \left(\frac{x(e^x + 1)}{e^x - 1} - 4 \right), \tag{7}$$

where

$$x = \frac{hc\nu}{kT},$$

4. Consequences of the solid angle difference on μ and y upper limits

The linearized models of cosmological distortions can be compared to the linearized error due to a 5% difference between the two solid angles: the most interesting case is the Compton distortion one, because the upper limit on y is the most

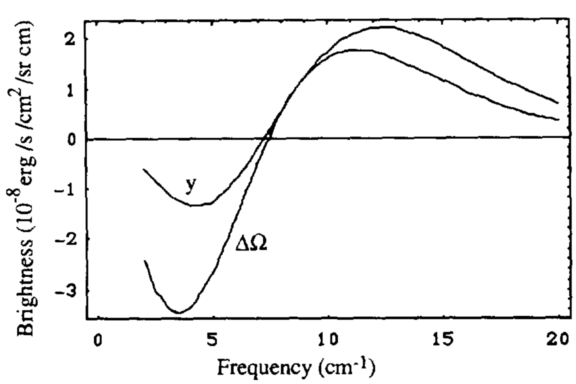

Fig. 3. Error due to 5% difference between the two solid angles and a Compton distortion with $y = 4 \times 10^{-5}$ (i.e. the y-value that minimizes the mean square difference between the two functions). The pure blackbody spectrum corresponds to the x-axes.

stringent. A direct comparison of Compton distortion with $y = 2.5 \times 10^{-5}$ to the solid angle difference error with $q = 2 \times 10^{-4}$ shows that, in the frequency range 2–20 cm^{-1} (considered by Mather et al. [5]), the former is almost entirely masked by the latter (Fig. 2). A simple least-mean-square fit between the Compton distortion and the solid angle difference error to the parameter y and q can be performed with the following results:

(i) a solid angle difference error with $q = 2 \times 10^{-4}$ is fitted by a Compton distortion with $y = 4 \times 10^{-5}$ (Fig. 3)

(ii) a Compton distortion with $y = 2.5 \times 10^{-5}$ is fitted by a solid angle difference error with $q = 9 \times 10^{-5}$, corresponding to a difference between the two solid angles of 2–3% (Fig. 4).

In both cases the maximum difference in the Wien region between the two functions is $\sim 7 \times 10^{-9}$ ergs $s^{-1} cm^{-2} sr^{-1}$ cm, just ~0.006% of the peak brightness. So, to place an upper limit of $y < 2.5 \times 10^{-5}$ a difference between the two solid angles well below 2% is necessary. On the other hand, with a 5% difference it is not possible to constrain y below $\sim 4 \times 10^{-5}$. Similar considerations show that to gain one order of magnitude in the upper limit on y, i.e. to get $y_{max} \sim 3 \times 10^{-6}$, the maximum difference between the two solid angles allowed is ~0.3%.

A comparison of the Bose–Einstein distortion with $\mu = 3.3 \times 10^{-4}$ to the error due to a 5% difference between the two solid angles (Fig. 5) shows the following situation: the former is greater than the latter in the lowest frequency range (2–10 cm^{-1}), while in the range 10–20 cm^{-1} there is the opposite situation. So, a Bose–Einstein distortion with $\mu = 3.3 \times 10^{-4}$ could have been seen in the lowest frequency region even if the difference between the two solid angles was of the order of 5%. However, such a difference would mask a Bose–Einstein distortion with $\mu \sim < 3 \times 10^{-4}$, therefore the upper limit placed by Mather et al. [5] seems to be just reasonable.

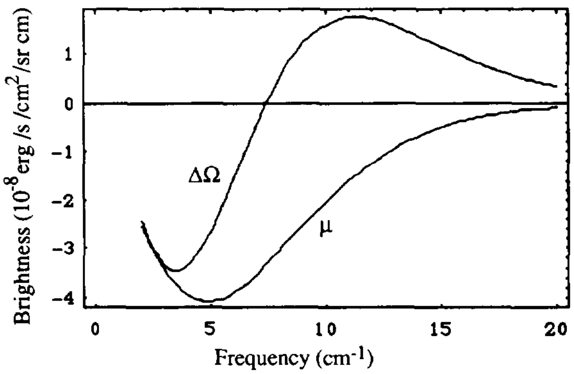

Fig. 5. Error due to a 5% difference between the two solid angles compared to a Bose–Einstein distortion with $\mu = 3.3 \times 10^{-4}$. The μ-distortion could have been seen only in the lowest frequency range. The pure blackbody spectrum corresponds to the x-axes.

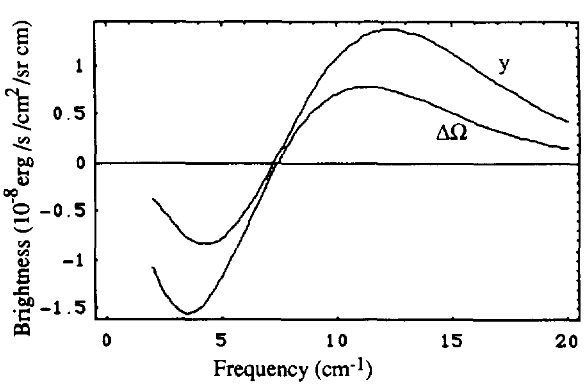

Fig. 4. Compton distortion with $y = 2.5 \times 10^{-5}$ and the error due to a difference between the two solid angles of the order of 2% (i.e. with the value of q that minimizes the mean square difference between the two functions). The pure blackbody spectrum corresponds to the x-axes.

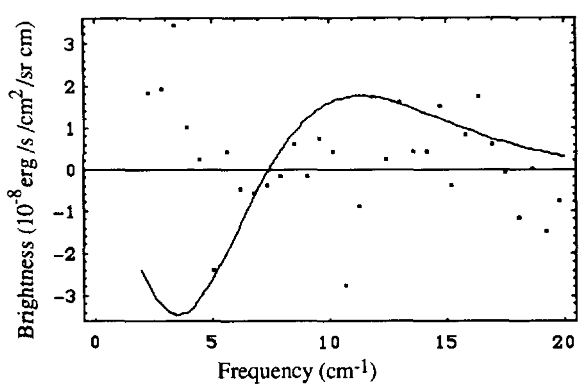

Fig. 6. Error due to a 5% difference between the two solid angles compared to the deviations found by Mather et al. from the fit used to place the upper limits on μ and y. The error bars are not reported. The pure blackbody spectrum corresponds to the x-axes.

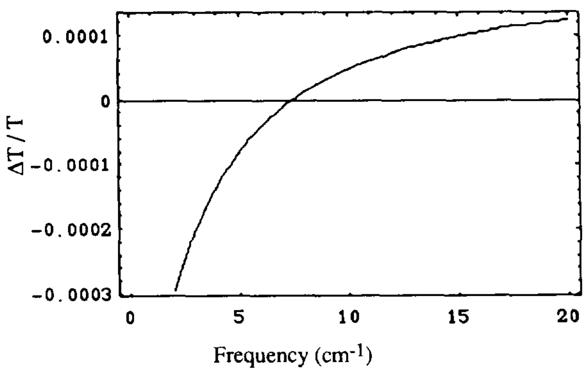

Fig. 7. Relative error in the measurement of CMBR temperature due to a difference between the two solid angles of 5%. The maximum value of ΔT is about 0.8 mK, lower than the error of 10 mK given by Mather et al.

It is worth observing that a 5% solid angle difference error is almost within the deviations found by Mather et al. [5] from the fit used to obtain the upper limits on μ and y and reported in Section 3 (Fig. 6). Therefore, if an effective 5% difference exists, such deviations are not significative.

5. Error on temperature measurement due to the solid angle difference

A 5% difference between the two solid angles introduces an error on CMBR temperature measurement. The analytic form of this error can be obtained by the simple relation

$$\Delta T = \left(\frac{\partial I(\nu, T)}{\partial T}\right)^{-1} \Delta I \tag{8}$$

which, in the present case, leads to

$$\frac{\Delta T}{T} = \frac{1 - e^{-x}}{x} \frac{\Delta I}{I}. \tag{9}$$

FIRAS data are well fitted by a Planck function with $T = 2.726$ K. The maximum error introduced by a $\Delta\Omega/\Omega \simeq 0.05$ is about 0.8 mK (Fig. 7), entirely negligible compared to the systematic error on temperature measurement given by Mather et al. [5] of 10 mK.

6. Summary

The new upper limit on y by FIRAS is not reasonable if a 5% difference between the two solid angles of the optics is considered. To place the limit $y < 2.5 \times 10^{-5}$ it is necessary at least the condition $\Delta\Omega/\Omega < 0.02$ and to gain a further order of magnitude the condition $\Delta\Omega/\Omega < 0.002$ is needed. The upper limit on μ seems to be more reasonable, but the assumed 5% difference does not allow to place any significative lower upper limit.

The effects of a 5% difference between the two solid angles on temperature measurement are entirely within the systematic error given by Mather et al. [5].

In the Fixsen's et al. paper [7] on calibration of FIRAS, problems that arise from a difference between the solid angles of the optics are not considered at all: more detailed information on the performances of the instrument should be provided.

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